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Recursive calculation abilities in agrammatic aphasia

A pilot study

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aphasia

ABSTRACT

In a pilot study we found that agrammatic aphasia restricted the complexity of feasible arithmetical operations but left intact the ability of estimating quantities relative to one another as well as the ability to construct recursive sequences of figures and operations. Recursive numerical sequences and recursive operations were retained in the form of schemata or constructions. We argue for a common recursion module in the human mind that may be accessible for representations of arithmetical constructions, whereas the representations of linguistic constructions may be detached from that module in the case of Broca's aphasia.

1. Introduction

We conducted a pilot study in which we investigated possible impairments of recursive operations in arithmetical tasks. We started from the assumption that some prerequisites of arithmetical operations are sensitivity to structural relationships and the ability to perform recursive operations (cf. Hauser et al. 2002; Spelke & Tsivkin 2001; Krajcsi 2006). Therefore, we tested a healthy and a Broca's aphasic participant for their abilities to count and to carry out arithmetical operations. Beyond their comprehension of figures and the ability to estimate quantities, we were primarily interested in how much the participants retained of their sensitivity to structural features of arithmetical operations, especially the infinite recursion of sequences of figures.

2. Agrammatic aphasia and calculation

2.1. Varley et al. (2005) studied agrammatic aphasics' calculation abilities. Their motivation was that the grammars of natural languages and of arithmetical expressions exhibit some parallelism. These parallels include recursion and structure dependence. For instance, the computation of the correct result of numerical expressions involving subtraction or division: $(5 - 10; 10 - 5; 5 \div 10; 10 \div 5)$ or the ability to follow the bracketing of an expression $[5 \times (6 + 2)]$ requires awareness of the structural properties of the given expression. Similarly, recursive rule application allows for the derivation of a potentially infinite number of outputs from a finite set of constituents. This property is found both in natural language and the language of arithmetic (e.g., *The man that has a hat that has a brim that has a...*; $2 + 3 + 5 + 7 + \ldots + \ldots$). The interdependence of language and arithmetic can also be seen in devices like the "multiplication table", a way of encoding mathematical facts in a verbal form and storing the result in one's long-term memory. The content thus stored can be accessed with no computation load when it is needed in the solution of novel calculation tasks, minimizing the required overall computation load. The interaction between arithmetical procedures and the activation of learned verbal information leads to the hypothesis that the operation of multiplication can be especially sensitive to aphasics' linguistic limitations (Lemer et al. 2003).

In the case of unimpaired persons, during the execution of numerical tasks, a bilateral network of cerebral regions is activated to mirror operations of calculating the quantity of objects, sounds or other entities. Several studies have detected activity in the language centers of the left hemisphere when the task was to perform exact calculations with symbolic expressions (Cohen et al. 2000; Friederici et al. 2011; Friedrich & Friederici 2013). Among others, this was found in multiplication by one-digit numbers where the use of verbally encoded information is crucial: the frontal "linguistic" areas, including Broca's area, were found to be activated (Dehaene et al. 1999; van Harskamp & Cipolotti 2001; Delazer et al. 2003). On the other hand, in cases of aphasic language impairment, concomitant problems in calculation abilities have been attested (e.g., Cohen et al. 2000).

However, an alternative approach is also conceivable. Although arithmetical operations are carried out by processes that are also required for lexical and grammatical operations, by the time ontogenesis reaches adulthood, the architecture of the mature mind reserves a niche for counting that is independent of language. Some studies claim that in counting tasks, stronger activation shows up in the right hemisphere (in the intraparietal sulcus) than on the left side (Butterworth 1999; Dehaene et al. 2003). Some functional cerebral imaging techniques seem to suggest that "linguistic areas" are not active in calculation tasks (Pesenti et al. 2000; Zago et al. 2001). Some accounts claim that in developmental and acquired language impairments, linguistic and mathematical abilities may be dissociated, that is, they do not form a single system of abilities (e.g., Ansari et al. 2003). But such dissociations do not exclude the possibility that subsystems of the grammar and the lexicon do support calculation performance, even in the case of language impairment.

2.2. The studies by Varley et al. (2005) and Zimmerer & Varley (2010) were groundbreaking in that they focused on the issue of whether recursion and sensitivity to the peculiarities of hierarchical structure were parallel/interdependent properties of linguistic and arithmetical procedures.

Varley et al. (2005) studied three agrammatic aphasic persons. All three were university graduates, one of them had been a professor of mathematics until he was afflicted with aphasia. According to the test results, the calculation procedures of the three persons, including recursive operations and sensitivity to hierarchical structures, had remained intact, while they were moderate agrammatic aphasics in terms of both computer tomography (CT) results and performance in status tests. They performed relatively well on lexical comprehension and synonym finding. But they exhibited severe impairment of linguistic-syntactic abilities and produced guessing-level results in grammaticality decisions with respect to written sentences. They also showed asyntactic sentence comprehension and guessing-level results in understanding "reversible" sentences. Their spontaneous speech production consisted of broken phrases or constituents thereof. Varley and her colleagues administered meticulous subtests on the abilities of reading numbers as symbols and of identifying (mathematical) operators, these being prerequisites to performing well on calculation tests. Out of the three persons, only one was able to use lexical names of numbers in speech, while the other two were not. On the other hand, the calculation of quantities and their ratios turned out to be unimpaired for all of them.

The calculation tests were pen-and-paper-based and consisted of eight subtests: (i) estimating the relative positions of quantities along a vertical line; (ii) addition, subtraction, multiplication and division operations on integers, then (iii) addition and subtraction of fractions; (iv) multiplication both on the basis of the multiplication table and beyond it; (v) inverting an operation yielding a positive number into one yielding a negative number;

(vi) creating infinite sets of numbers; (vii) operations involving bracketing, where in some cases the brackets were syntactic in the sense that simply performing the operations left-to right would not give the correct result (e.g., $36 \div (3 \times 2)$), while in other cases the brackets were non-syntactic (e.g., $(3 \times 3) - 6$); and persons were also asked to (viii) generate bracketing (they received sequences of figures and operators with the instruction that they should insert brackets in several different manners and then calculate the results accordingly). We will return to the details of these subtests in our discussion of their Hungarian adaptations. In what follows, we will compare the performance of a healthy person and an aphasic participant. Varley et al.'s participants achieved good results in each of the subtests; in some cases they performed without a single error. The results show the mutual independence of structure-based linguistic vs. arithmetical operations within a given cognitive architecture. Although all persons were agrammatic aphasics, they applied syntactic principles in arithmetic appropriately.

Varley et al. (2005) and Zimmerer & Varley (2010) proposed two types of explanations of the interrelationship of the syntax of language and the syntax of arithmetic. According to one, the two systems work independently of each another, and the impairment of one does not need to concern the other. According to the other explanation, there is a shared syntactic system that underlies both language and arithmetic, but arithmetical processing may directly access this system without translating the expressions into a linguistic form first.

3. Participants

The aphasic participant was C, a 31-year-old right-handed man, 17 years of schooling, an engineer. He was assigned to aphasia type on the basis of CT results, the Western Aphasia Battery (WAB) tests (Kertesz 1982) and the Token test (De Renzi & Vignolo 1962). The WAB test and the Token test were adapted to Hungarian by Osmanné Sági (1991; 1994). The CT showed an isochemic stroke at the left arteria cerebri media. On the basis of the results of the Western Aphasia Battery (WAB), he was a Broca's aphasic with severe agrammatism, his Aphasia quotient (AQ) equalled 56 (in healthy participants: 93.8 or above; the maximum is 100). In the Token test he achieved 14 scores (healthy subjects above 32; the maximum is 36).[1] According to the CT results and the results in the WAB and Token tests, C exhibited the typical symptoms of severe agrammatic Broca's aphasia. C participated in our earlier investigation on the capacity of recursive sentence embedding. In those experiments C was not able to produce responses containing recursive sentence embedding, he gave only some simple, short, fragmented answers (Bánréti et al. 2016). He was severely impaired in producing recursive syntactic structures.

The healthy subject was Z, a 42-year-old right-handed man with 16 years of schooling, a teacher.

4. Materials and methods

To test our subjects' performance on arithmetic, we administered a variety of tasks based on Varley et al. (2005), a total of seven subtests. For details see the Appendix.

For the estimation task, they had to mark the approximate positions of 20 numbers (presented to them in a random order) along a 20 cm vertical line (number line) of which the two ends were marked as 0 and 100, respectively. The task probed into the degree of limitation of the subjects' utilization of quantity concepts. (The task sheets can be found in the Appendix.) The response was taken to be correct if the marking provided by the subject was within 5 mm from the proper value point. Next, addition (12 items), subtraction (12 items), multiplication (9 items), and division tasks (16 items) followed. The correct results were positive integers in all cases.

[1] The Western Aphasia Battery (WAB) uses a kind of standard protocol. Spontaneous speech is evaluated for articulation, fluency, content and presence of paraphasias. Comprehension is tested with yes or no questions, pointing commands, and one to three step commands. Naming is evaluated for objects, object parts, body parts, and colors. Repetition is requested for single words to complex sentences. The level of adequacy for reading and writing is also tested. Five subtests on fluency, information, comprehension, repetition, and naming impairment are classified from 0 to 10. The maximum result of each subtest is 10 points each. Accordingly, aphasia can be classified into global aphasia, Broca's aphasia, Wernicke's aphasia, transcortical motor, transcortical sensory conduction aphasia, and anomic aphasia types. For instance, in Broca's aphasia fluency ranges from 0 only to 4 points, comprehension ranges from 4 to 10, repetition is under 8 points and naming ranges from 0 only to 8. Aphasia quotient (AQ) shows the severity of aphasia. AQ is calculated by the addition of scores of the subtests and this sum is multiplied by two. The maximum is 100. Normal subjects score an AQ of 93.8 or above. An AQ around 50 shows a severe degree of aphasia, cf. John et al. (2017).

In the inversion task, two-digit numbers had to be subtracted or divided in a random order, such that first a smaller number had to be subtracted from a larger one (respectively, a larger number had to be divided by a smaller one) yielding a positive integer (e.g., $72-26; 60 \div 12$), then the other way round (yielding a negative number for subtraction and a fraction for division, e.g., $26 - 72; 12 \div 60$). All this was done in three instances.

In the bracketing resolution task, there were expressions involving syntactic bracketing (8 items) in which, if the subject followed just the linear order of operations without taking the brackets into consideration, the result would be incorrect, as in $36 \div (3 \times 2)$; and there were also expressions with non-syntactic bracketing (3 items) in which the correct result is obtained whether or not the brackets are taken into consideration, as in $12 \times (6 \times 7)$. Among the syntactic items, there were single and double pairs of brackets. In the latter case, another operation was embedded as a term of the main operation. While single bracketing occurred in the left term or in the right term double bracketing invariably occurred in the second term (8 items).

In the bracket generation task, the subject had to generate bracketing on sequences of four numbers linked by operators such that different ways of bracketing should yield different results, e.g., $(6+2) \times 5 + 8 = 48; 6 + (2 \times 5) + 8 = 24; 6 + 2 \times (5+8) = 32; (6+2) \times (5+8) = 104$; etc. The use of brackets in calculation tasks is taken to be an instruction for recursive operations as in these cases one or more terms of an expression are themselves results of a recursively embedded operation.[2]

In the infinity task, the subjects had to generate sequences of numbers. They had to find numbers larger than one but smaller than two, then after each response a number that is larger than the previous answer but still smaller than two, and so on – keeping on increasing the values without reaching the number two.

[2] Arithmetic operations have a default order: if only additions and subtractions are involved, their order does not matter. If a division or multiplication is one of the operations required and an addition or subtraction is the other, it is always the division/multiplication that comes first. Between division and multiplication, the order has to be signaled by bracketing, e.g., $(6 \div 3) \times 2 = 4$ but $6 \div (3 \times 2) = 1$. In complex expressions, it is the expression within the brackets that has to be calculated first; and within a pair of brackets, multiplication and division enjoy applicational precedence over addition and subtraction. For instance: $3 \times (20–\underline{5 \times 2}) = 3 \times (20–10) = 3 \times 10 = 30$. The default order (division/multiplication first) can be overridden by bracketing: $(6+2) \times 5 + 8 = 48; 6 + (2 \times 5) + 8 = 24; 6 + 2 \times (5+8) = 32$.

5. Results

5.1. Normal participant

The calculation tasks did not represent any difficulty for the normal participant except that he required a relatively long concentration of attention. The sporadically occurring errors may be due to that factor.

Tables 1 and 2 show percentages of errors in each task (n = all calculations performed by the subject, 0: percentage of erroneous calculations if no errors were made).

Table 1: Results of tasks in the arithmetical test in percentages of errors: normal participant

Subject	Estimation task $n = 20$	The four basic operations $n = 12$ /12 /10 /10				Subtraction and its inversion (yielding a negative number) $n = 6$	Division and its inversion (yielding a fraction) $n = 6$	Infinity $n = 11$
		$+$	$-$	$\times$	$\div$			
Z	1.9	0	0	0	10	0	0	0

Table 2: Results of bracketing tasks in percentages of errors: normal participant

Subject	Bracketing operations			
	Single bracketing $n = 20$	Double bracketing $n = 8$	Generation of bracketing $n = 25$	Resolution of bracketing $n = 25$
Z	0	0	0	0

5.2. Agrammatic aphasic participant

Tables 3 and 4 show percentages of errors in each task (n = all calculations performed by the subject, 0: percentage of erroneous calculations if no errors were made).

Table 3: Results of tasks in the arithmetical test in percentages of errors: aphasic participant

Subject	Estimation task $n = 20$	The four basic operations $n = 12$ / 10 / 10 / 9				Subtraction and its inversion (yielding a negative number) n = 6	Division and its inversion (yielding a fraction) $n = 6$	Infinity $n = 10$
		$+$	$-$	$\times$	$\div$			
C agrammatic aphasic subject	4.6	0	0	0	13	50	66.6	0

Table 4: Results of bracketing tasks in percentages of errors: aphasic participant

Subject	Bracketing operations			
	Single bracketing $n = 9$	Double bracketing $n = 3$	Generation of bracketing $n = 9$	Resolution of bracketing $n = 9$
C agrammatic aphasic subject	42.8	33.3	0	100

In the task involving the number line, C made few mistakes in localizing given numerical values along the line, the deviation amounted to 4.6%. (A 20 cm vertical line was at the subjects' disposal, so 2.5% difference meant 5 mm, C's average deviation – above 5 mm – only about 4 mm above the tolerance threshold). We can conclude that the notion of quantities represented by figures was unimpaired in both subjects.

With respect to the four basic operations, he was successful in addition and in subtraction. Here we found correct results for single-digit, two-digit, and three-digit numbers. In case of multiplication, only that of single-digit and two digit terms were done correctly, while no calculations with multiple digits were carried out at all. The division of a two-digit number by a one-digit number was correct, while C performed only one of the division tasks of three-digit numbers correctly; out of nine cases, he gave the wrong result in one case and gave a result that was roughly correct but not to the last decimal value in another.

Half of the inversion tasks yielded the wrong result in subtractions, and more than half of them in the case of divisions. In the case of negative results, C signaled by [–] that he would get a negative number, but most results were wrong. In the cases of dividing a smaller number by a larger one, he overlooked only in one out of three cases that the result would not be an integer.

The knowledge that sequences of numbers may be infinite was retained. He found numbers larger than one but smaller than two correctly; he gave several correct solutions and recognized the rule.

In bracketing operations, the order of operations in tasks involving a single pair of brackets was correct; the final results were not always correct due to calculation errors. In operations involving multiple brackets (embeddings), the partial calculations and the order of operations were correct, but the final result could not always be given.

In the last task, C was able to generate bracketing (embedding), he inserted brackets at different places in each case but failed to calculate the final results. In that task, he also used double bracketing.

6. Discussion

6.1. Impairments in linguistic resources

C had difficulties in verbalizing his calculations, but he was capable of self-monitoring. He showed several types of impairments in linguistic resources available for arithmetic operations, especially impairments in the lexical access of numerals. During the calculations, the digits were spontaneously read aloud, of which "9" was mistakenly read as "8" but in each case he corrected himself to "9". C hesitated typically at the verbal markers indicating place values; for example, he said: *nyolc... száz... őőő nem!... nyolc... VAN... hat* 'eight... hundred... hmm... no!... eight... TY... six'. In the end, he was always able to produce the correct name of the digit; he could encode the visual input into verbal form.

In his calculations, the names of the signs "+" and "−" were produced (called "plus" and "minus"). The verbal equivalents (names) of the division and multiplication were not used spontaneously. The operations $A \times B$ and $C \div D$ were called "A and B", "C and D". C did not use the words "division", "multiplication" either in nominal or in verbal functions (i.e., the phrases "A multiplied by B", and "C divided by D" were never realized). At the same time, the symbolic signs of multiplication and division were understood and the operations indicated by them could be performed, and their results were often accurate, even if not always. In other words, he performed the operations of multiplication, division, addition and subtraction without lexically accessing the exact names of those operations, except for using the expressions "plus" and "minus" for addition and subtraction, respectively.

C did not produce the names of the fractional numbers, he did not say "one third" or "two sixth", for digits like 1/3, 2/6, etc. but he called them *1 tört 3, 2 tört 6* '1 fraction 3, 2 fraction 6', etc. when he was asked to report on how he counted. At the same time, the results of addition and subtraction of the fractions were correct; he used the value of the common denominator of the fractions independently and correctly.

C did not use the term "bracket" either during the silent reading of the tasks or in the completion of the tasks containing brackets. He was able to produce the sequence and embedding of counting operations following the hierarchy required by the brackets.

6.2. Effects related to aphasic limitation

We can identify several different effects related to aphasic limitation. The first effect is that of complexity; in particular, the complexity of operations to be performed in each task. C was able to perform addition and subtraction without errors, while in division and multiplication three digit terms were avoided, inversion resulting in a negative number and the resolution of all bracketed formulae came with high error rates and the resolution of formulae generated by the subject himself proved to be even more difficult (100% error rate). Such robust effect of complexity is of course not surprising under the circumstances of severe agrammatic aphasia.

The second effect involves the participant's ability to estimate quantities within a given domain. C's error rate was only 4.6%. His ability in certain calculation procedures is more severely impaired than his ability to estimate quantities within a domain. C did not commit any errors with respect to the relative order of the quantities along the number line; the error rates werre due to cases in which they marked the points of the number line more than 5 mm off target. (C's average deviation – above 5 mm – was only about 4 mm). We can conclude that the notion of quantities represented by figures was unimpaired in the aphasic participant.

The third effect was the ability to generate sequences of numbers in a recursive manner. This was probed into by the task requiring the production of infinite sequences of numbers (Appendix VIII), and by the task asking for the generation of bracketing (Appendix VII) – including multiple bracketing – of the same series of numbers in several different ways. In those two tasks, C performed without error. He was able to produce sequences of numbers and sequences of operations that recursively contained other sequences of numbers and operations, respectively. However, C was not able to do the actual calculation on task (viii). He did not produce erroneous results but rather deemed the calculation of the value of the formula he had produced himself to be too difficult and gave up without trying.

In sum, the agrammatic aphasia did affect (limit) the complexity of calculations but left the ability intact to estimate relative distances of quantities and the ability to create recursive sequences of numbers or operations. The latter was done in terms of schemas/constructions, the calculations yielded concrete numerical end products were avoided.

7. Conclusion

The dissociations outlined above are interesting especially in view of the fact that in earlier studies we found strong limitations of linguistic-syntactic recursion in Broca's aphasia (Bánréti et al. 2016). These observations are now complemented by the finding that, in the case of arithmetical operations, albeit complexity effects show up similarly, the relative estimation of quantities and the ability of generating recursive sequences of numbers and operations can be retained in Broca's aphasia. The arithmetic operations by the aphasic person show limitations, but these do not concern the basic operations themselves, only their more complex versions. The difficulties in accessing the verbal linguistic resources that are useful for counting may lead to errors or confusion in more complex calculations but do not make them inaccessible. Recursive numerical sequences and recursive operations are retained in the form of schemata or constructions.

Some patterns of linguistic and arithmetic expressions have similarities, such as recursiveness and structure dependency. A moderate aphasic condition exhibits strong limitations in those linguistic (primarily syntactic and lexical) capacities, but arithmetical operations show a much better state preserving basic operations. This provides arguments for a model in which linguistic and arithmetic processes are separated, and counting may be kept separate from language in adult age. In this model recursive arithmetical operations can be carried out inspite of linguistic impairments.

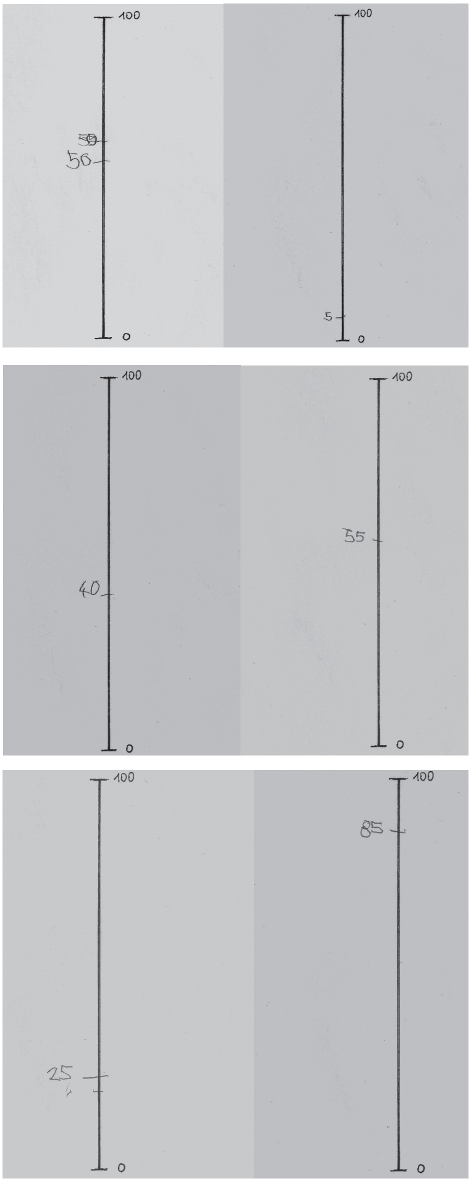
Our results support the model proposed by Zimmerer and Varley (2010) that posits a module of recursive operations in the human mind that are shared (among others) by linguistic and arithmetical performance. This common recursion module may be accessible for representations of arithmetical constructions, whereas the representations of linguistic constructions may be detached from it in the case of Broca's aphasia. Varley et al. (2005) point out that in adult age[3] arithmetic can be sustained without the grammatical and lexical resources of the language.

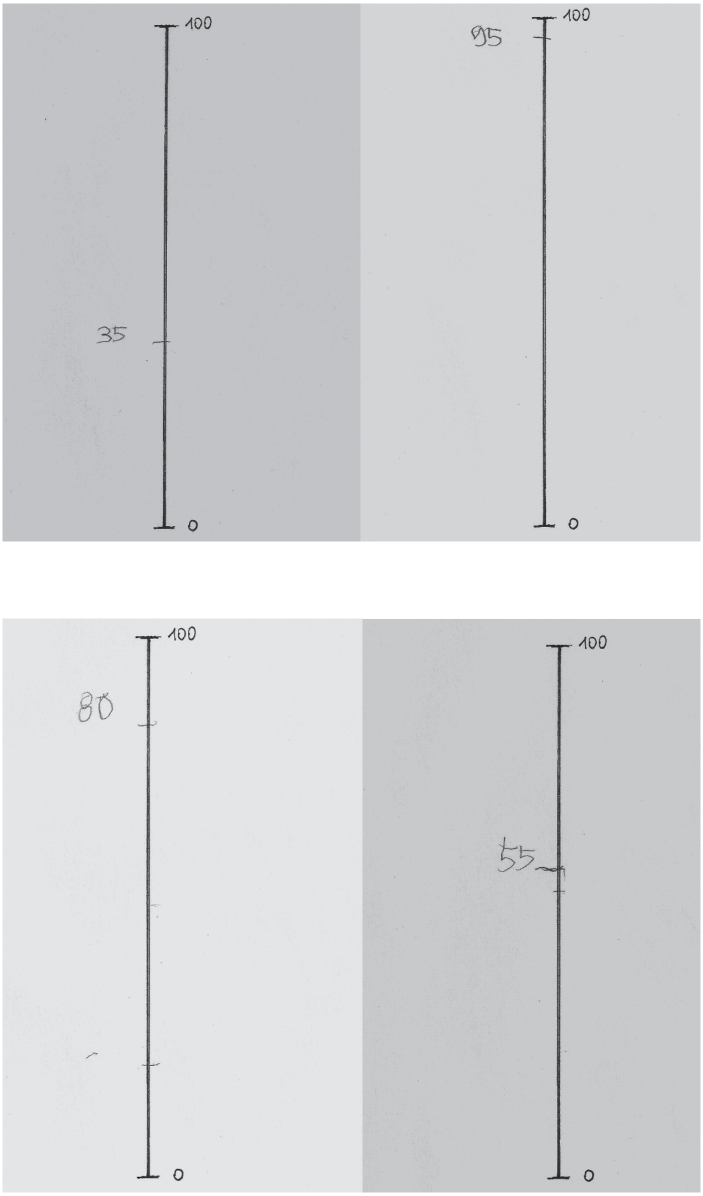
[3] Varley et al. (2005, 6) state: "Number words may be important in children's acquisition of numerical concepts and their digital, orthographic, phonological, and sensory representations. Similarly, language grammar might provide a 'bootstrapping' template to facilitate the use of other hierarchical and generative systems, such as mathematics. However, once these resources are in place, mathematics can be sustained without the grammatical and lexical resources of the language faculty. [...] grammar may thus be seen as a co-opted system that can support the expression of mathematical reasoning, but the possession of grammar neither guarantees nor jeopardizes successful performance on calculation problems".

Appendix

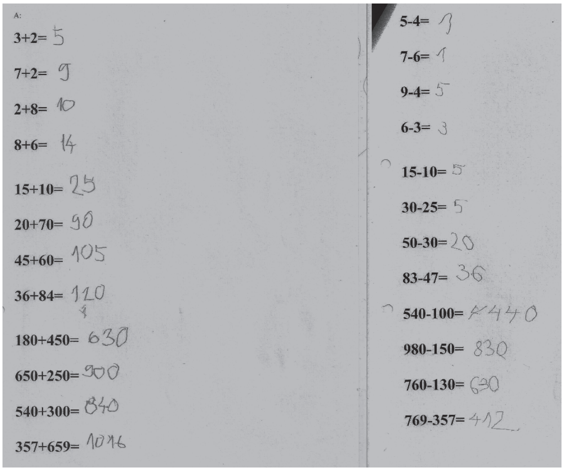
Aphasic participant: some examples

I. Estimation tasks



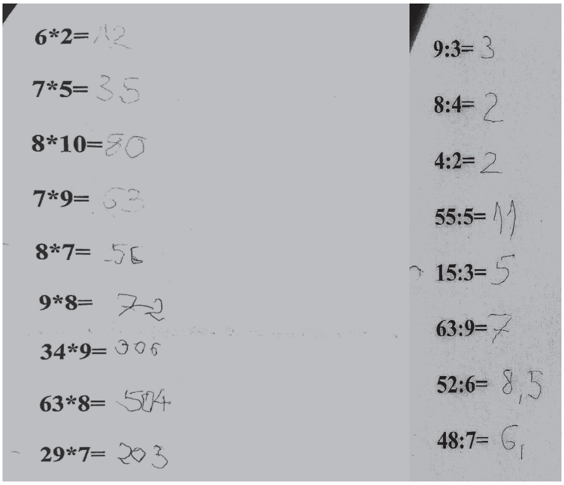

100
35
0
100
95
0
100
80
0
100
55
0

II. Basic operations: addition, subtraction, multiplication, division tasks



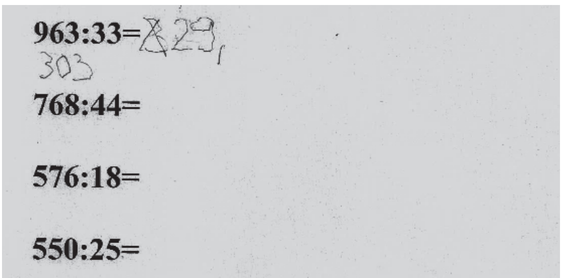
A:
3+2= 5
7+2= 9
2+8= 10
8+6= 14
15+10= 25
20+70= 90
45+60= 105
36+84= 110
180+450= 630
650+250= 900
540+300= 840
357+659= 1016

5-4= 1
7-6= 1
9-4= 5
6-3= 3
15-10= 5
30-25= 5
50-30= 20
83-47= 36
540-100= ~~4~~440
980-150= 830
760-130= 630
769-357= 412



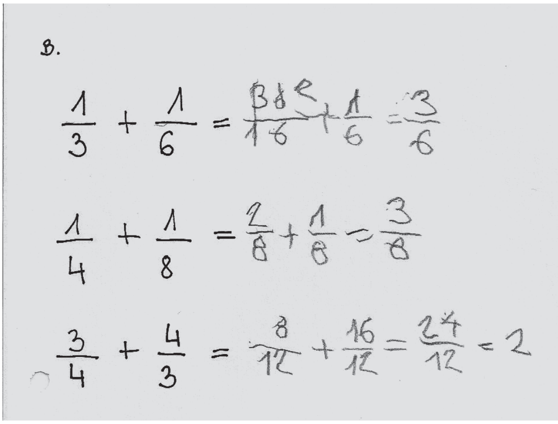
6*2= 12
7*5= 35
8*10= 80
7*9= 63
8*7= 56
9*8= 72
34*9= 306
63*8= 504
29*7= 203

9:3= 3
8:4= 2
4:2= 2
55:5= 11
15:3= 5
63:9= 7
52:6= 8,5
48:7= 6,



963:33= ~~8~~29,
303
768:44=
576:18=
550:25=

III. Addition of fractional numbers, finding their common denominators

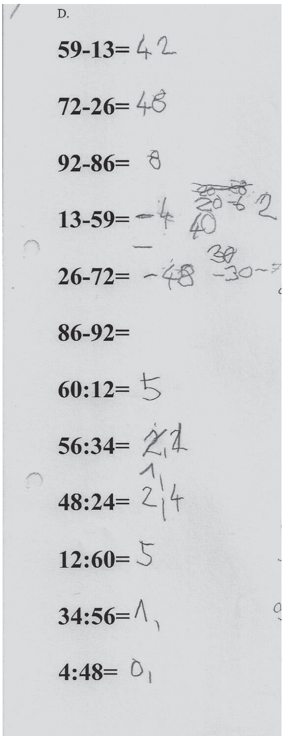

B.

$\frac{1}{3} + \frac{1}{6} = \frac{\cancel{3 d e}}{\cancel{16}} + \frac{1}{6} = \frac{3}{6}$

$\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$

$\frac{3}{4} + \frac{4}{3} = \frac{8}{12} + \frac{16}{12} = \frac{24}{12} = 2$

IV. "Inversion" tasks


D.

59-13= 42

72-26= 48

92-86= 8

13-59= −4 40

26-72= −48 −30

86-92=

60:12= 5

56:34= 2,1

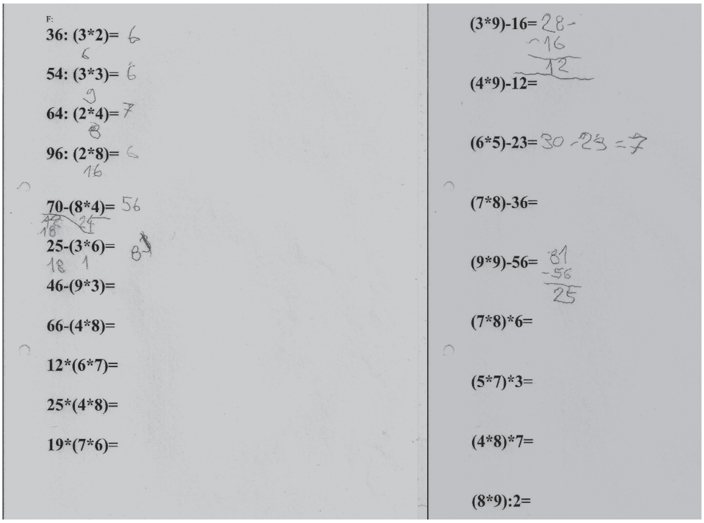
48:24= 2,4

12:60= 5

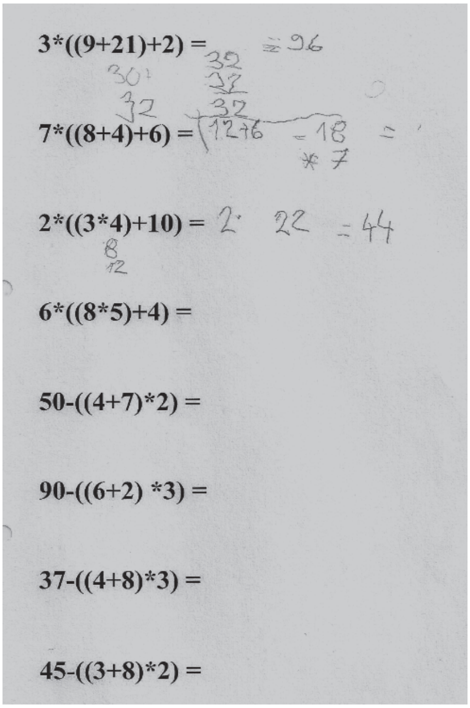
34:56= 1,

4:48= 0,

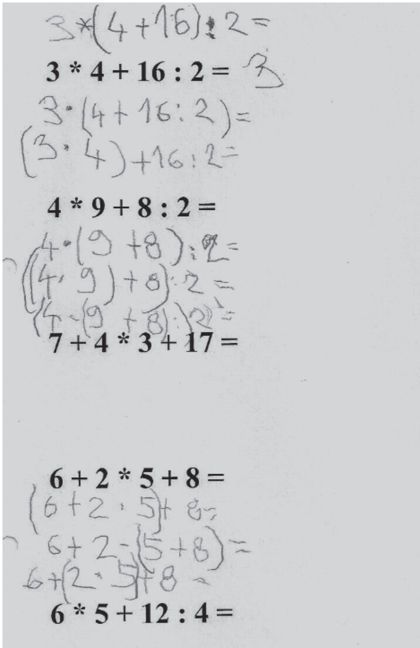
V. Single bracketing tasks


F:
36: (3*2)= 6
6
54: (3*3)= 6
9
64: (2*4)= 7
8
96: (2*8)= 6
16
70-(8*4)= 56
25-(3*6)= 8
18 1
46-(9*3)=
66-(4*8)=
12*(6*7)=
25*(4*8)=
19*(7*6)=
(3*9)-16= 28-
-16
12
(4*9)-12=
(6*5)-23= 30 -23 =7
(7*8)-36=
(9*9)-56= 81
-56
25
(7*8)*6=
(5*7)*3=
(4*8)*7=
(8*9):2=

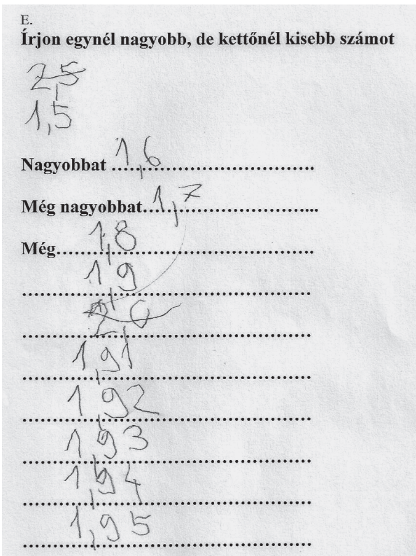
VI. Double bracketing tasks


3*((9+21)+2) = =96
30
32
32
32
32
7*((8+4)+6) = 12+6 =18 =
*7
2*((3*4)+10) = 2· 22 =44
8
12
6*((8*5)+4) =
50-((4+7)*2) =
90-((6+2) *3) =
37-((4+8)*3) =
45-((3+8)*2) =

VII. Generation of bracketing tasks

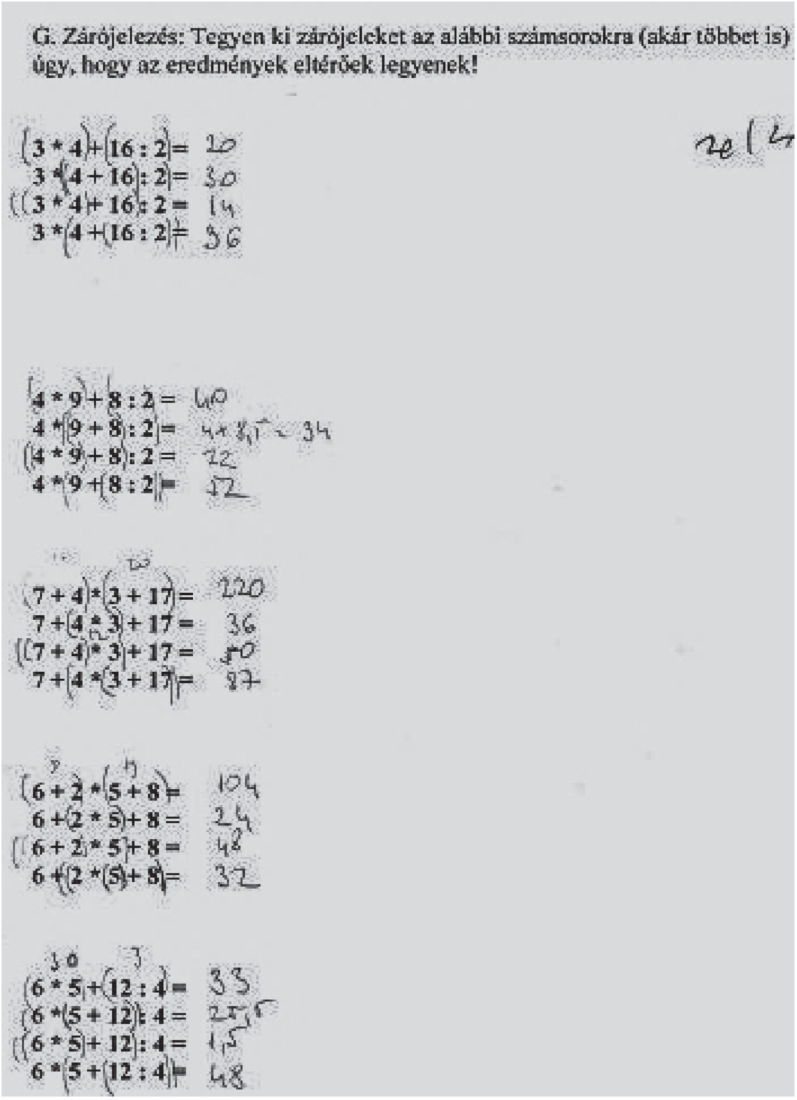


VIII. "Infinity" task



Normal participant: some examples

IX. Generation and resolution of bracketing task



G. Zárójelezés: Tegyen ki zárójeleket az alábbi számsorokra (akár többet is) úgy, hogy az eredmények eltérőek legyenek!

(3 * 4)+(16 : 2)= 20
3 *(4 + 16): 2)= 30
((3 * 4)+ 16): 2 = 14
3 *(4 +(16 : 2)) = 36

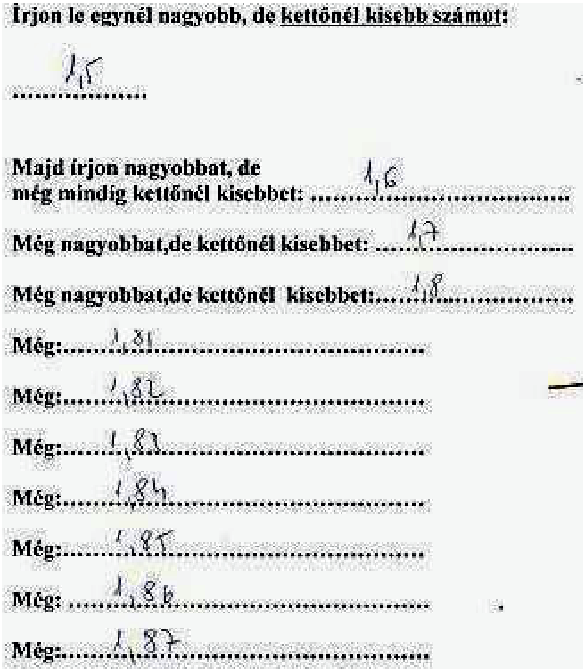
(4 * 9)+(8 : 2) = 40
4 *(9 + 8): 2)= 4×8,5 = 34
((4 * 9)+ 8): 2 = 22
4 *(9 +(8 : 2)) = 52

(7 + 4)*(3 + 17)= 220
7 +(4 * 3)+ 17 = 36
((7 + 4)* 3)+ 17 = 50
7 +(4 *(3 + 17)) = 87

(6 + 2)*(5 + 8)= 104
6 +(2 * 5)+ 8 = 24
((6 + 2)* 5)+ 8 = 48
6 +(2 *(5)+ 8) = 32

(6 * 5)+(12 : 4)= 33
(6 *(5 + 12)): 4 = 25,5
((6 * 5)+ 12): 4 = 1,5
6 *(5 +(12 : 4)) = 48

X. "Infinity" task



Írjon le egynél nagyobb, de kettőnél kisebb számot:

1,5

Majd írjon nagyobbat, de még mindig kettőnél kisebbet: 1,6

Még nagyobbat,de kettőnél kisebbet: 1,7

Még nagyobbat,de kettőnél kisebbet: 1,8

Még: 1,81

Még: 1,82

Még: 1,83

Még: 1,84

Még: 1,85

Még: 1,86

Még: 1,87

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